Midterm 1

for Math 308, Winter 2018

NAME (last - first): ______________________________

- Do not open this exam until you are told to begin. You will have 50 minutes for the exam.
- This exam contains 5 questions for a total of 50 points in 9 pages.
- You are allowed to have one double sided, handwritten note sheet and a non-programmable calculator.
- Show all your work. With the exception of True/False questions, if there is no work supporting an answer (even if correct) you will not receive full credit for the problem.

Do not write on this table!

Question	Points	Score
1	6	
2	4	
3	15	
4	12	
5	13	
Total:	50	

Statement of Ethics regarding this exam

I agree to complete this exam without unauthorized assistance from any person, materials, or device.

Signature: ______________________________ Date: ____________

Question 1. (6 points) Decide whether the following statements are true or false. For this you don't need to show any work.

(a) [1 point] If the augmented matrix of a linear system has more rows than columns, then the system is inconsistent.

○ True ○ False

(b) [1 point] If u, v_1, v_2 are three vectors in $\mathbb{R}^3$ such that u is in the span(v_1, v_2) then span$\{u, v_1, v_2\} = \mathbb{R}^3$.

○ True ○ False

(c) [1 point] A 1-to-1 linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ is also onto.

○ True ○ False

(d) [1 point] A matrix in reduced echelon form might have rows of zeros.

○ True ○ False

(e) [1 point] Given any set of $m > n$ vectors $u_1, \ldots, u_m \in \mathbb{R}^n$, span$(u_1, \ldots, u_m) = \mathbb{R}^n$.

○ True ○ False

(f) [1 point] If the linear transformation $T_A : \mathbb{R}^n \to \mathbb{R}^m$ is onto, the linear system $Ax = b$ is consistent for any $b \in \mathbb{R}^m$.

○ True ○ False

Question 2. (4 points) For any of the following question, give an explicit example. If it is not possible write *NOT POSSIBLE*. You don't need to write any justification for this question.

(a) [1 point] Give an example of an inconsistent linear system with more variables than equations.

(b) [1 point] Give an example of four vectors in $\mathbb{R}^4$ whose span is NOT $\mathbb{R}^4$ and none of them is a multiple of another.

(c) [1 point] Give an example of a linear transformation $T_A : \mathbb{R}^2 \to \mathbb{R}^3$ that is 1-to-1.

(d) [1 points] Give an example of two matrices A and B such that $AB \neq BA$.

Question 3. (15 points) Consider the following linear system:

$$\begin{aligned} x_1 + + 6x_3 + 2x_4 &= 13 \\ x_1 + x_2 + 9x_3 + 3x_4 &= 20 \\ x_2 + 6x_3 + x_4 &= 13 \end{aligned}$$

(a) [2 points] Write down the associated augmented matrix.

(b) [6 points] Compute the REF of the matrix using the Gauss-Jordan algorithm.

(c) [3 points] Using the previous part, write down the solution set in vector form. What is the dimension of the solution set?

(d) [4 points] Given your previous computation explain why or why not

$$\begin{pmatrix} 13 \\ 20 \\ 13 \end{pmatrix} \in \text{span}\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 6 \\ 9 \\ 6 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \}$$

Question 4. (12 points) Consider the following 3 vectors in $\mathbb{R}^3$

$$u_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \quad u_2 = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \quad u_3 = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$$

(a) [4 points] Show that $\text{span}\{u_1, u_2, u_3\} = \mathbb{R}^3$.

(b) [4 points] Express the vector $v = \begin{pmatrix} 3 \\ 5 \\ 11 \end{pmatrix}$ as a linear combination of the vectors u_1, u_2, u_3 above.

(c) [4 points] Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation such that

$$T(u_1) = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \qquad T(u_2) = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \qquad T(u_3) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Using the properties of linear transformations compute $T(v)$ where v is the same vector of the previous part.

Question 5. (13 points) Consider the following matrix and linear transformation

$$A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \qquad T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -x_1 + x_2 + x_3 \\ 2x_1 - 6x_2 - 8x_3 \\ 2x_2 + x_3 \\ 2x_1 - 6x_2 \end{pmatrix}$$

(a) [2 points] Determine a matrix B such that $T = T_B$, i.e. such that $T(x) = B \cdot x$.

(b) [2 points] Determine domain and codomain of the linear transformation T_A (the linear transformation associated to A) and T_B.

(c) [6 points] For every composition that make sense compute domain, codomain and associated matrix of $T_A \circ T_B$ and $T_B \circ T_A$.

(d) [3 points] For every composition that make sense determine whether $T_A \circ T_B$ and $T_B \circ T_A$ are 1-to-1 and/or onto and/or invertible.